

8.5 Power spectrum of signal and noise

Suppose the noise is additive; call it x and the signal, s . Then, using upper case for the transforms, the transform must be $S + X$. The power spectrum is this times its complex conjugate, so

$$(S + X)(S^* + X^*) = |S|^2 + |X|^2 + 2\text{Re}(S^*X).$$

The first term on the right-hand side is the power spectrum of the signal. The second is the spectrum of the noise, which (for Gaussian noise) is a chi-square random variable. The last term contains X , which will be Gaussian (the real and imaginary parts separately and independently Gaussian), scaled up by S . Re denotes the real part of all this. So in the power spectrum, the noise level will in general depend on the power spectrum – the noise is no longer additive. If $S \rightarrow 0$ at some large value (the smoothing of a point-spread function often achieves this) then the level of the spectrum gives a guide to the noise – if it is white and additive in the original data.